EMPIRICAL RESULTS IN FINITE POPULATION SAMPLING

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A. Statement of Problem

i. Given: A finite population of units $U_1, \ldots, U_N$ with $y_1, \ldots, y_N$ unknown before sampling and $x_1, \ldots, x_N$, constants, known before sampling. We wish to estimate

$$T = \sum_{j=1}^{N} y_j.$$

ii. How to Estimate T

a. In the traditional theory (see, for example, Horwitz and Thompson (1952)) one takes a simple random sample of size n and uses, perhaps,

$$\frac{N}{n}\sum_s y \quad \text{or} \quad \frac{\sum_s y}{\sum_s x}\sum_{i=1}^{N} x_i.$$

b. Prediction Theory (Least Squares Approach) $y_1, \ldots, y_N$ are realized values of random variables $Y_1, \ldots, Y_N$. $E(Y_i)=f(x_i)$ and Var $(Y_i)=\sigma^2 v(x_i)$. For example, if $E(Y_i)=\beta x_i$, Var $(Y_i)=\sigma^2 x_i$, then the linear estimator, $\hat{T}$ for

$T=\sum_{j=1}^{N} Y_j$ which minimizes $E(T-\hat{T})^2$ and satisfies

$E(T-\hat{T})=0$ is $\hat{T}=\frac{\sum_s y}{\sum_s x}\sum_{i=1}^{N} x_i$, i.e., the ratio estimator.

iii. Bias Within the Prediction Context

Within the prediction approach to estimation in finite populations an estimator $\hat{T}$ for T is said to be unbiased for T if $E(\hat{T}-T)=0$. Now, under a model such that $E(Y_i)=\beta_0+\beta_1 x_i$, we have

$$E\left(T-\frac{\sum_s y}{\sum_s x}\sum_{i=1}^{N} x_i\right)=\beta_0\left(\frac{N\sum_s x_i - n\sum_{i=1}^{N} x_i}{\sum_s x_i}\right)$$

which is, in general, nonzero unless

$$\frac{\sum_{i=1}^{N} x_i}{N}=\frac{\sum_s x_i}{n}$$

, that is, unless the units are chosen in such a way that the average on the auxiliary variable x within the sample is equal to the average on x within the entire finite population. Such a sample is said to be balanced on the first moment of x.

B. Objective of Paper

It is the aim of this paper to demonstrate the importance of balanced samples. This will be accomplished by showing that the error in estimating a known total for a specific population is much smaller when the samples are balanced than when samples departing significantly from balance are used. It will also be noted that measures of error for the estimate of the total will have more validity when balanced samples are used. This result holds for a wide variety of estimators.

C. The Population and Sampling Procedure

The population studied consisted of 150 Negro males, ages 10, 11, and 12. On each individual the three variables, weight, height, and chest circumference were measured. Samples of sizes 5, 10, and 30 were selected.

For each sample size, samples composed of the largest and smallest units on the variable height as well as samples approximately balanced on height were chosen. The balanced samples were obtained by ranking the units on height and selecting the proper number of units equally spaced on the ranks of height. For example, a sample approximately balanced on height of size 30 was obtained by selecting the 5th largest, 10th largest, 15th largest, etc., down to the smallest unit on the variable height. A variety of estimators were compared with regard to their ability to estimate the total weight of the 150 Negro males. For each estimator studied, the associated estimate of variance was also tabulated.

The Estimators

The estimators will now be listed. Some of the estimators use two auxiliary variables, x_0 and x_1. For this study the variable weight played the role of y, height the role of x_1, and chest circumference the role of x_0.

Olkin's Estimator

Olkin's estimator with two auxiliary variables x_0 and x_1 is given by

$$\hat{T}_{OL}=\omega_1\frac{\sum_s y_j}{\sum_s x_{j1}}\sum_{j=1}^{N} x_{j1}+\omega_2\frac{\sum_s y_j}{\sum_s x_{j0}}\sum_{j=1}^{N} x_{j0} \qquad (1)$$

where $\omega_1+\omega_2=1$. The weights ω_1, ω_2 depend upon the sample through a 2x2 matrix A, where

$$a_{11}=\sum_s\left(y_j-\frac{\sum_s y_j}{\sum_s x_{j1}}x_{j1}\right)^2, \quad a_{22}=\sum_s\left(y_j-\frac{\sum_s y_j}{\sum_s x_{j0}}x_{j0}\right)^2 \text{ and}$$

$$a_{12}=a_{21}=\sum_s\left(y_j-\frac{\sum_s y_j}{\sum_s x_{j1}}x_{j1}\right)\left(y_j-\frac{\sum_s y_j}{\sum_s x_{j0}}x_{j0}\right).$$

Then $\underline{\omega}=(\omega_1,\omega_2)=\frac{e'A^{-1}}{e'A^{-1}e}$ where $e'=(1,1)$. A measure of uncertainty for (1) is given by

$$\hat{\sigma}_{OL}=\left[\frac{N^2}{ne'A^{-1}e}\right]^{1/2}. \qquad (2)$$

Least Squares Prediction Estimators

Now, under the model $Y_j=\beta_2 x_{j1}+\varepsilon_{j2}[v_2(x_{j1})]^{1/2}$ where β_2 is unknown but constant, $E(\varepsilon_{j2})=0$, and and $VAR(\varepsilon_{j2})=\sigma_2^2$; and an analogous model

$Y_j=\beta_1 x_{j0}+\varepsilon_{j1}[v_1(x_{j0})]^{1/2}$ where again β_1 is unknown but constant, $E(\varepsilon_{j1})=0$, and $VAR(\varepsilon_{j1})=\sigma_1^2$, the least squares theory of estimation in finite populations gives as the optimal estimator when $v_2(x_{j1})=x_{j1}$ and $v_1(x_{j0})=x_{j0}$ just the ratio estimators based on x_1 and x_0 respectively. Expressions (3) - (8) list the appropriate estimators under these two models along with measures of uncertainty provided by least squares theory and traditional finite sampling theory. Let

$\bar{x}_i(\bar{s})=\frac{1}{N-n}\sum_{\bar{s}} x_{ji}$, $\bar{x}_i(s)=\frac{1}{n}\sum_s x_{ji}$ and $\bar{x}_i(P)=\frac{1}{N}\sum_{j=1}^{N} x_{ji}$, $i=0,1$. Then

$$R(x_1)=\frac{\sum_s y_j}{\sum_s x_{j1}}\sum_{j=1}^{N} x_{j1}, \qquad (3)$$

$$\hat{\sigma}_{R(x_1)} = [\frac{N^2}{n}(1-\frac{n}{N})\frac{1}{n-1}\sum_s (y_j - \frac{\sum_s y_j}{\sum_s x_{j1}} x_{j1})^2/x_{j1} \quad (4)$$
$$\cdot \frac{\bar{x}_1(\tilde{s})}{\bar{x}_1(s)} \bar{x}_1(P)]^{1/2} \quad \text{(least squares)},$$

$$\hat{\sigma}_{R(x_1)} = [\frac{N^2}{n}(1-\frac{n}{N})\frac{1}{n-1}\sum_s (y_j - \frac{\sum_s y_j}{\sum_s x_{j1}} x_{j1})^2]^{1/2} \quad (5)$$
(traditional),

$$R(x_0) = \frac{\sum_s y_j}{\sum_s x_{j0}} \sum_{j=1}^{N} x_{j0}, \quad (6)$$

$$\hat{\sigma}_{R(x_0)} = [\frac{N^2}{n}(1-\frac{n}{N})\frac{1}{n-1}\sum_s (y_j - \frac{\sum_s y_j}{\sum_s x_{j0}})^2/x_{j0} \quad (7)$$
$$\cdot \frac{\bar{x}_0(\tilde{s})}{\bar{x}_0(s)} \bar{x}_0(P)]^{1/2} \quad \text{(least squares)},$$

$$\hat{\sigma}_{R(x_0)} = [\frac{N^2}{n}(1-\frac{n}{N})\frac{1}{n-1}\sum_s (y_j - \frac{\sum_s y_j}{\sum_s x_{j0}})^2]^{1/2} \quad (8)$$
(traditional).

When $v_2(x_{j1}) = v_1(x_{j0}) = 1$, least squares gives the following estimators and measures of uncertainty:

$$\hat{T}_1 = \sum_s y_j + \hat{\beta}_2 \sum_{\tilde{s}} x_{j1} \text{ where } \hat{\beta}_2 = \frac{\sum_s y_j x_{j1}}{\sum_s x_{j1}^2} \quad (9)$$

$$\hat{\sigma}_{\hat{T}_1} = [\hat{\sigma}_2^2 [\frac{(\sum_{\tilde{s}} x_{j1})^2}{\sum_s x_{j1}^2} + (N-n)]]^{1/2} \quad (10)$$

where $\hat{\sigma}_2^2 = \frac{1}{n-1}\sum_s (y_j - \hat{\beta}_2 x_{j1})^2$.

$$\hat{T}_2 = \sum_s y_j + \hat{\beta}_1 \sum_{\tilde{s}} x_{j0}, \quad (11)$$

$$\hat{\sigma}_{\hat{T}_2} = [\hat{\sigma}_1^2 [\frac{(\sum_{\tilde{s}} x_{j0})^2}{\sum_s x_{j0}^2} + (N-n)]]^{1/2} \quad (12)$$

where $\hat{\beta}_1 = \frac{\sum_s y_j x_{j0}}{\sum_s x_{j0}^2}$ and $\hat{\sigma}_1^2 = \frac{1}{n-1}\sum_s (y_j - \hat{\beta}_1 x_{j0})^2$.

Another least squares model whose inclusion is quite natural is

$Y_j = \beta_1 x_{j0} + \beta_2 x_{j1} + \varepsilon_j [v(x_{j0}, x_{j1})]^{1/2}$. The variance function $v(x_{j0}, x_{j1}) = 1$ was specified, not through any reasoning process, in fact, but by default, since the variation of y over the x_0, x_1 plane is difficult to determine. Under this model, we have through least squares theory,

$$\hat{T}_3 = \sum_s y_j + \hat{\beta}_1 \sum_{\tilde{s}} x_{j0} + \hat{\beta}_2 \sum_{\tilde{s}} x_{j1} \text{ where} \quad (13)$$

$$\hat{\beta}_1 = \frac{1}{\sum_s x_{j0}^2 \sum_s x_{j1}^2 - (\sum_s x_{j1}x_{j0})^2} \cdot [\sum_s x_{j1}^2 \sum_s y_j x_{j0} - \sum_s x_{j1}x_{j0} \sum_s y_k x_{j1}]$$

$$\hat{\beta}_2 = \frac{1}{\sum_s x_{j0}^2 \sum_s x_{j1}^2 - (\sum_s x_{j1}x_{j0})^2} \cdot [\sum_s x_{j0}^2 \sum_s y_j x_{j1} - \sum_s x_{j1}x_{j0} \sum_s y_j x_{j0}]$$

and a measure of uncertainty is

$$\hat{\sigma}_{\hat{T}_3} = [\hat{\sigma}^2 [\frac{\sum_s (x_{j1}\sum_{\tilde{s}} x_{j0} - x_{j0}\sum_{\tilde{s}} x_{j1})^2}{\sum_s x_{j0}^2 \sum_s x_{j1}^2 - (\sum_s x_{j1}x_{j0})^2} + (N-n)]]^{1/2} \quad (14)$$

where $\hat{\sigma}^2 = \frac{1}{n-2}[\sum_s (y_j - \hat{\beta}_1 x_{j0} - \hat{\beta}_2 x_{j1})^2]$.

Finally, since the expansion estimator plays such an important role in sampling theory, being a favorite traditional estimator and the optimal estimator in the least squares theory when the sampling is balanced, it was included along with a measure of uncertainty specified by the least squares model $Y_j = \mu + \varepsilon_j$, under which it is optimal for any sample.

$$\hat{T}_4 = \frac{N}{n}\sum_s y_j \quad (15)$$

$$\hat{\sigma}_{\hat{T}_4} = [\frac{N(N-n)}{n(n-1)}\sum_s (y_j - \bar{y}(s))^2]^{1/2} \quad (16)$$

<u>Srivastava's Estimators</u>

Srivastava [1971] proposes a class of estimators, three of which are considered in this study. Each estimator is an adjusted expansion estimator and they are given by (17) - (19)

$$\hat{T}_1 = \frac{N}{n}(\sum_s y_j)\exp\{\sum_{i=0}^{1}\theta_i \log u_i\} \quad (17)$$

$$\hat{T}_2 = \frac{N}{n}(\sum_s y_j)\exp\{\sum_{i=0}^{1}\theta_i (u_i - 1)\} \quad (18)$$

$$\hat{T}_3 = \frac{N}{n}(\sum_s y_j)\{\sum_{i=0}^{1}\omega_i \exp\{(\theta_i/\omega_i)\log u_i\}\}. \quad (19)$$

where $\omega_0 + \omega_1 = 1$, $u_i = \frac{\bar{x}_i(s)}{\bar{x}_i(P)}$ $i=0,1$ and

$$\underset{\sim}{\theta} = \binom{\theta_0}{\theta_1} = -A^{-1}\underset{\sim}{b}.$$

In this study the values $\omega_0 = \omega_1 = 1/2$ were selected. For two auxiliary variables A has elements

$$a_{11} = \frac{\sum_s (x_{j0} - \bar{x}_0(s))^2 (\bar{y}(s))^2}{(\bar{x}_0(s))^2 \sum_s (y_j - \bar{y}(s))^2},$$

$$a_{22} = \frac{\sum_s (x_{j1} - \bar{x}_1(s))^2 (\bar{y}(s))^2}{(\bar{x}_1(s))^2 \sum_s (y_j - \bar{y}(s))^2} \quad \text{and}$$

$$a_{12} = a_{21} = \frac{\sum_s (x_{j0} - \bar{x}_0(s))(x_{j1} - \bar{x}_1(s))(\bar{y}(s))^2}{\bar{x}_0(s)\,\bar{x}_1(s)\sum_s (y_j - \bar{y}(s))^2}$$

The vector $\underset{\sim}{b}$ has elements

$$b_1 = \frac{\sum_s (y_j - \bar{y}(s))(x_{j0} - \bar{x}_0(s))\,\bar{y}(s)}{\sum_s (y_j - \bar{y}(s))^2\,\bar{x}_0(s)},$$

$$b_2 = \frac{\sum_s (y_j - \bar{y}(s))(x_{j1} - \bar{x}_1(s))\bar{y}(s)}{\sum_s (y_j - \bar{y}(s))^2 \bar{x}_1(s)} .$$

A measure of uncertainty for (17) - (19) is given by

$$\hat{\sigma}_S = [\frac{N(N-n)}{n(n-1)} \sum_s (y_j - \bar{y}(s))^2 (1 - b' A^{-1} b)]^{1/2}. \quad (20)$$

(See page 405 of Srivastava [1971].)

Singh's Estimators

We consider four estimators discussed by Singh [1967]. The estimators along with their measures of uncertainty are given by (21) - (28).

$$\hat{T}_1 = \frac{N}{n}(\sum_s y_j) \frac{\sum_s x_{j1} \sum_1^N x_{j0}}{\sum_1^N x_{j1} \sum_s x_{j0}} \quad (21)$$

$$\hat{\sigma}_{\hat{T}_1} = [\frac{(N-n)}{Nn} (\sum_{j=1}^N y_j)^2 [C_y^2 + C_{x_0}^2 - 2C_y C_{x_0} \rho_{y,x_0} + C_{x_1}^2 + 2C_y C_{x_1} \rho_{y,x_1} - 2C_{x_0} C_{x_1} \rho_{x_0,x_1}]]^{1/2} \quad (22)$$

$$\hat{T}_2 = \frac{N}{n} (\sum_s y_j) \frac{\sum_s x_{j0} \sum_1^N x_{j1}}{\sum_1^N x_{j0} \sum_s x_{j1}} \quad (23)$$

$$\hat{\sigma}_{\hat{T}_2} = [\frac{(N-n)}{Nn} (\sum_{j=1}^N y_j)^2 [C_y^2 + C_{x_1}^2 - 2C_y C_{x_1} \rho_{y,x_1} + C_{x_0}^2 + 2C_y C_{x_0} \rho_{y,x_0} - 2C_{x_0} C_{x_1} \rho_{x_0,x_1}]]^{1/2} \quad (24)$$

$$\hat{T}_3 = \frac{n}{N} (\sum_s y_j) \frac{\sum_1^N x_{j0} \sum_1^N x_{j1}}{\sum_s x_{j0} \sum_s x_{j1}} \quad (25)$$

$$\hat{\sigma}_{\hat{T}_3} = [\frac{(N-n)}{Nn} (\sum_{j=1}^N y_j)^2 [C_y^2 + C_{x_0}^2 - 2C_y C_{x_0} \rho_{y,x_0} + C_{x_1}^2 - 2C_y C_{x_1} \rho_{y,x_1} + 2C_{x_0} C_{x_1} \rho_{x_0,x_1}]]^{1/2} \quad (26)$$

$$\hat{T}_4 = (\frac{N}{n})^3 \frac{\sum_s y_j \sum_s x_{j0} \sum_s x_{j1}}{\sum_1^N x_{j0} \sum_1^N x_{j1}} \quad (27)$$

$$\hat{\sigma}_{\hat{T}_4} = [\frac{(N-n)}{Nn} (\sum_{j=1}^N y_j)^2 [C_y^2 + C_{x_1}^2 + 2C_y C_{x_1} \rho_{y,x_1} + C_{x_0}^2 + 2C_y C_{x_0} \rho_{y,x_0} + 2C_{x_0} C_{x_1} \rho_{x_0,x_1}]]^{1/2}$$

For each of the measures of uncertainty $(\sum_{j=1}^N y_j)^2$ was estimated by the square of the expansion estimator. A term like C_y^2 was estimated by

$$\hat{C}_y^2 = \frac{n^2}{(n-1)} \frac{\sum_s (y_j - \bar{y}(s))^2}{(\sum_s y_j)^2}$$

and ρ with any pair of subscripts by the appropriate Pearson produce moment correlation coefficient based on observations in the sample.

Discussion

Tables I and II list certain parameters of the population and the samples chosen from the population. Note, for example, how closely the sample mean $\bar{h}(s)$ for samples 9-13 approximates the population mean of 57.17 for the variable height. For extreme samples, for example 1 and 7, the balance is not so good. Table III gives actural error in estimating the known total of 12416 lbs. along with the ratio of absolute error to estimated standard error for Srivias-tava's extimators. For example, column 3 of table III gives 21.9 as the ratio of the absolute error, 5428, to the calculated value of expression (20), for sample I. Note the inadequacy of (20), as a measure of true error in this case. The results on standard error for Singh's estimators are similar to those observed in table III. Tables IV -VI compare balanced samples with samples based on the extreme units of the variables height.

Summary of Results

For samples departing from balance, that is, those based on the largest and smallest units, quite poor estimates of the true total and unrealistic measures of error are observed, particularly for small sample sizes.

The estimators of Singh and Srivastava performed poorly for samples based on the extreme units and many of their estimators perform poorly on balanced samples. The error estimates for Singh's and Srivastava's estimators, in general, perform well only for balanced samples.

The results suggest that sampling plans which insure balance are preferred to those under which extreme departures from balance are possible such as simple random sampling.

TABLE I

PARAMETERS OF POPULATION WHERE N IS THE TOTAL NUMBERS OF UNITS AND h, w, AND c, DESIGNATE HEIGHT, WEIGHT AND CHEST CIRCUMFERENCE RESPECTIVELY

$N = 150$

$\sum_1^N w_j = 12416$ lbs.

$\sum_1^N h_j = 8575$ ins., $\sum_1^N h_j/150 = 57.17$, $\sum_1^N h_j^2/150 = 3276$

$\sum_1^N c_j = 3998$ ins., $\sum_1^N c_j/150 = 26.66$, $\sum_1^N c_j^2/150 = 715$

TABLE II

SAMPLE NUMBER AND SIZE - SAMPLING CONFIGURATION AND FIRST AND SECOND SAMPLE MOMENTS FOR THE TWO AUXILIARY VARIABLES

Sample Number	n	Sampling Confuguration	$\bar{h}(s)$	$\bar{h}^2(s)$	$\bar{c}(s)$	$\bar{c}^2(s)$
1	5	1,. . . ,5	64.2	4123	28.6	817
2	5	10,40,...130	57.6	3324	26.7	713
3	5	16,46,..,136	57.2	3274	26.5	715
4	5	15,45,..,135	57.2	3274	26.5	707
5	5	14,44,..,134	57.2	3282	26.9	730
6	5	20,50,..,140	56.8	3235	25.5	652
7	5	146, . .,150	51.0	2603	25.4	644
8	10	1,. . ., 10	63.1	3985	28.1	792
9	10	5,20,..,150	57.4	3304	26.3	694
10	10	7,22,..,140	57.3	3287	25.6	661
11	10	8,23,..,143	57.2	3280	26.6	710
12	10	9,24,..,144	57.1	3266	28.5	823
13	10	10,24,..,145	57.0	3258	26.5	704
14	10	141, . .,150	51.9	2698	25.2	636
15	30	1,. . ., 30	61.3	3759	28.1	796
16	30	5,10,..,150	60.0	3253	26.4	701
17	30	1,6,. .,146	57.4	3301	26.8	726
18	30	2,7,. .,147	57.3	3288	25.9	676
19	30	3,8,. ..,148	57.2	3276	26.7	713
20	30	4,9,. .,149	57.1	3263	27.4	757
21	30	121, . .,150	53.3	2846	25.6	658

TABLE III

ACTURAL ERROR AND ESTIMATE OF STANDARD ERROR ALONG WITH THE RATIO OF ABSOLUTE ERROR TO ESTIMATED STANDARD ERROR

	Srivastava's Estimators						
Sample Number	(17) $\hat{T} - T$	$\lvert\hat{T} - T\rvert/\widehat{S.E.}$	(18) $\hat{T} - T$	$\lvert\hat{T} - T\rvert/\widehat{S.E.}$	(19) $\hat{T} - T$	$\lvert\hat{T} - T\rvert/\widehat{S.E.}$	(20) $\widehat{S.E.}$
1	- 5428	21.9	- 5750	23.2	- 5105	20.6	248
2	- 756	3.8	- 757	3.5	- 765	3.6	217
3	+ 3.3	.0	+ 3.3	.0	+ 4.0	.0	124
4	+ 11.9	.1	+ 11.7	.1	+ 12.3	.1	101
5	+ 440	10.7	+ 439	10.7	+ 441	10.7	41
6	+ 126	.5	+ 106	.4	+ 149	.8	246
7	- 5285	18.7	- 2138	18.2	- 4168	14.8	282
8	- 1942	4.6	- 2145	5.0	- 1597	3.7	426
9	+ 391	2.0	+ 388	1.9	+ 399	2.0	200
10	- 393	1.4	- 400	1.4	- 386	1.3	291
11	+ 198	.5	+ 195	.5	+ 195	.9	381
12	- 275	.5	- 342	.6	- 108	.2	536
13	- 656	2.5	- 657	2.5	- 656	2.5	260
14	- 1702	6.4	- 1740	6.4	- 1697	6.4	264
15	- 939	4.3	- 1042	4.8	- 927	4.3	217
16	- 223	1.4	- 224	1.4	- 222	1.4	157
17	- 80	.5	- 80	.5	- 80	.5	171
18	- 71	.3	- 75	.4	- 64	.3	212
19	+ 185	.8	+ 185	.8	+ 185	.8	221
20	- 256	1.1	- 267	1.1	- 227	1.0	237
21	+ 450	3.3	+ 375	2.8	+ 483	3.6	136

TABLE IV

AVERAGE ABSOLUTE ERROR FOR BALANCED SAMPLES AND ABSOLUTE ERROR FOR EXTREME SAMPLES BASED ON LARGEST AND SMALLEST UNITS FOR EACH ESTIMATOR WITHIN EACH SAMPLE SIZE. COUNT OF ABSOLUTE ERRORS FALLING WITHIN TWO STANDARD ERRORS, UNDER "VARIANCE EVALUATION"

n	Sample Type	Least Squares (9)	Least Squares (11)	Srivastava (17)	Srivastava (18)
	BAL	438	333	268	263
5	EX { L	2970	3709	5428	5750
	S	1581	2233	5285	5138
	BAL	781	583	382	396
10	EX { L	2050	2728	1942	2145
	S	1792	2201	1702	1740
	BAL	357	200	163	166
30	EX { L	1792	1995	932	1042
	S	1098	1419	450	375
Variance Evaluation		14	15	12	12

TABLE V

AVERAGE ABSOLUTE ERROR FOR BALANCED SAMPLES AND ABSOLUTE ERROR FOR EXTREME SAMPLES BASED ON LARGEST AND SMALLEST UNITS FOR EACH ESTIMATOR WITHIN EACH SAMPLE SIZE. COUNT OF ABSOLUTE ERRORS FALLING WITHIN TWO STANDARD ERRORS, UNDER "VARIANCE EVALUATION"

n	Sample Type	Ratio Aux (h)	Ratio Aux (c)	Ratio Aux (c^2)	Ratio Aux (ch)
	BAL	476	334	281	335
5	EX { L	2968	3697	2691	1956
	S	1582	2250	1685	1012
	BAL	756	515	343	495
10	EX { L	2034	2711	1987	1306
	S	1789	2203	1561	1157
	BAL	350	181	178	173
30	EX { L	1719	1935	1189	989
	S	1106	1436	953	632
Variance		15 (5)	15 (5)	15	15
Evaluation		14 (4)	15 (4)		

n	Sample Type	Ratio Aux c^2h	Olkin's (1)	Least Squares (13)
	BAL	313	485	506
5	EX { L	1082	3789	3796
	S	362	3094	3096
	BAL	317	521	517
10	EX { L	669	2592	2614
	S	430	2222	2221
	BAL	192	225	212
30	EX { L	313	2090	2112
	S	97	1541	1530
Variance		19	14	
Evaluation				18

TABLE VI

AVERAGE ABSOLUTE ERROR FOR BALANCED SAMPLES AND ABSOLUTE ERROR FOR EXTREME SAMPLES BASED ON LARGEST AND SMALLEST UNITS FOR EACH ESTIMATOR WITHIN EACH SAMPLE SIZE. COUNT OF ABSOLUTE ERRORS FALLING WITHIN TWO STANDARD ERRORS, UNDER "VARIANCE EVALUATION"

n	Sample Type	Srivastava (19)	Expansion (15)	Singh (21)
5	BAL	272	476	335
	EX L	5105	4661	5679
	EX S	4168	2747	3343
10	BAL	349	766	526
	EX L	1597	3536	4284
	EX S	1697	2726	3139
30	BAL	156	358	189
	EX L	927	2739	2970
	EX S	483	1865	2173
Variance Evaluation		12	15	15

n	Sample Type	Singh (23)	Singh (25)	Singh (27)
5	BAL	614	333	812
	EX L	4080	1931	8390
	EX S	2111	1026	4208
10	BAL	1045	504	1037
	EX L	2822	1286	6156
	EX S	2371	1174	4127
30	BAL	527	173	543
	EX L	2511	969	4742
	EX S	1547	647	2956
Variance Evaluation		15	13	15

Bibliography

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